POPULATION OF PHILADELPHIA COUNTY

A. MATERIALS NEEDED

Worksheet with data table, calculator, ruler

B. OBJECTIVE

The student will use his knowledge of slopes of lines to calculate the average rates of change of the population of Philadelphia County, Pennsylvania. The student will also build upon his previous knowledge to discover relationships associated with the population function.

C. RATIONALE

The average rate of change of a function over an interval is defined to be the slope of the secant line over the interval. The formula being used is $\frac{f(b)-f(a)}{b-a}$, where the points $(a, f(a))$ and $(b, f(b))$ are the points on the curve at the ends of the interval. In this activity, the students will determine the average rate of change of a function over the domain of the function. The students will also concentrate on the meaning associated with the slope of the secant line in this context.

POPULATION OF PHILADELPHIA COUNTY

Objective: The student will use his knowledge of slopes of lines to calculate the average rates of change of the population of Philadelphia County, Pennsylvania. The student will also build upon his previous knowledge to discover relationships associated with the population function.

The population for Philadelphia County, Pennsylvania, was collected from the U. S. census reports beginning in 1900 and recorded in the table below. The web sites where the data were obtained are http://www.census.gov/population/www/censusdata/cencounts.html and http://quickfacts.census.gov/qfd/ .

Year	Population
1900	1293697
1910	1549008
1920	1823779
1930	1950961
1940	1931334
1950	2071605
1960	2002512
1970	1948609
1980	1688210
1990	1585577
2000	1517550

1. Sketch a graph of the data.

2. The data forms a function. Describe the function in words. Use as many descriptive words as possible to describe the characteristics of the function.

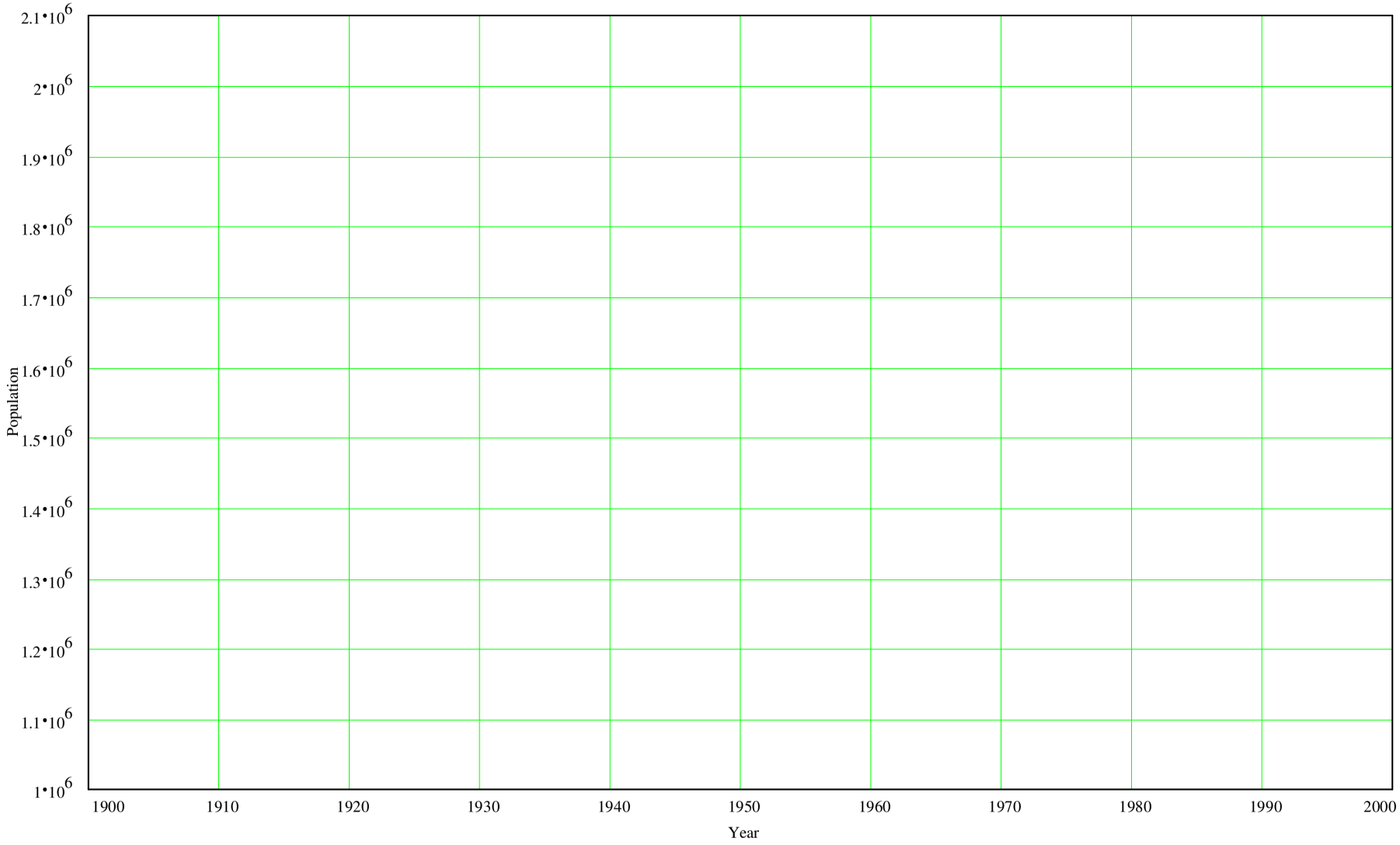
3. Using the graph of the population, draw a line from the first point (t = 1900) to the last point on the graph (t = 2000). What is the slope of the line? What is the meaning of the slope of the line?

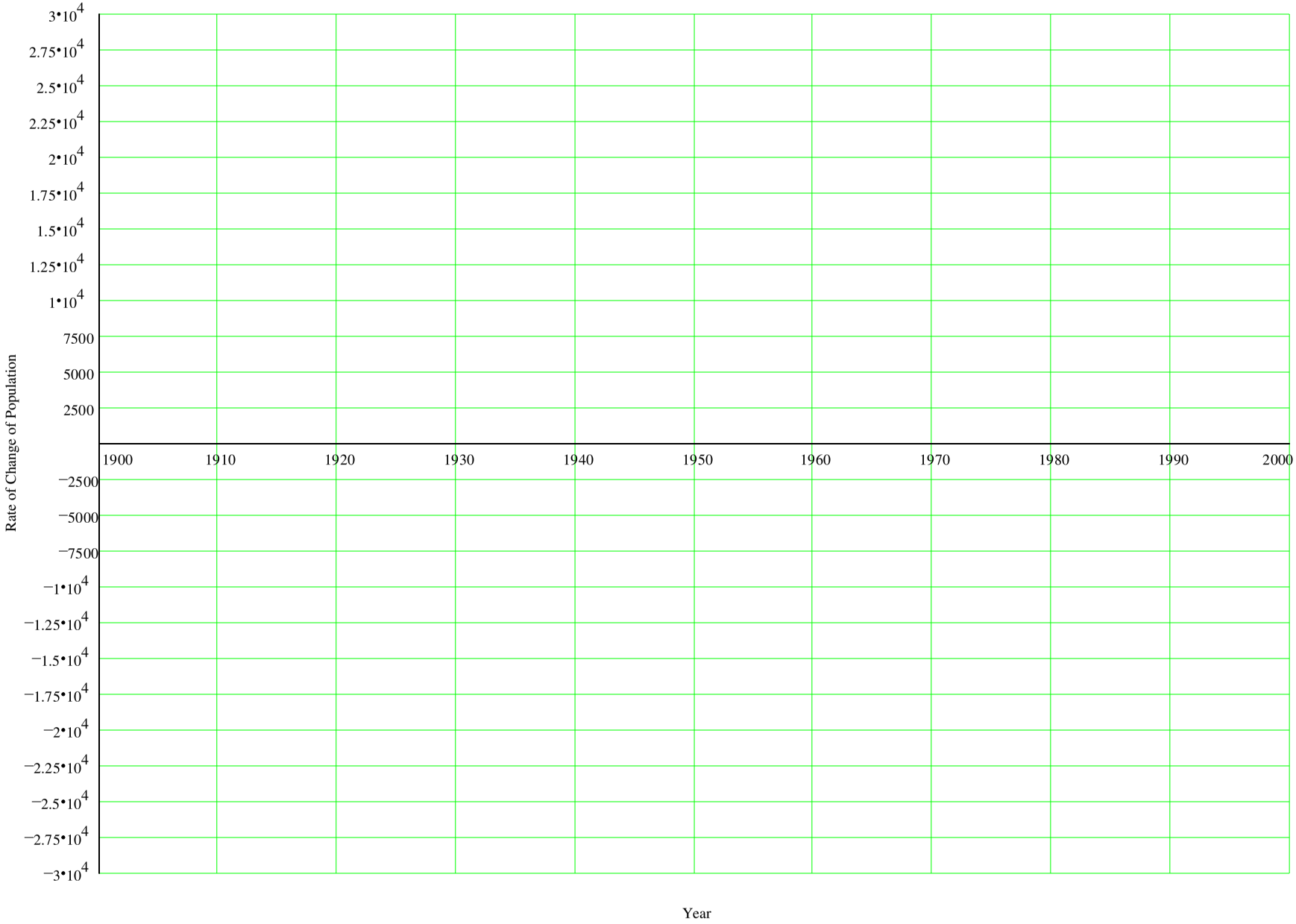
4. Compute the average rates of population change over the years indicated in the chart below.

Year	Population	Interval of Time	Average Rate of Change of Population
1900	1293697	1900 – 1910	
1910	1549008	1910 – 1920	
1920	1823779	1920 – 1930	
1930	1950961	1930 – 1940	
1940	1931334	1940 – 1950	
1950	2071605	1950 – 1960	
1960	2002512	1960 – 1970	
1970	1948609	1970 – 1980	
1980	1688210	1980 – 1990	
1990	1585577	1990 – 2000	
2000	1517550		

5. Plot the values of the average rates of change on a separate graph.

6. Some of the rates of change that were calculated and graphed are positive, and some of them are negative. Describe what is happening to the population function when these average rates are positive and when these average rates are negative.


2.1•10^6
2•10^6
1.9•10^6
1.8•10^6
1.7•10^6
1.6•10^6
1.5•10^6
1.4•10^6
1.3•10^6
1.2•10^6
1.1•10^6
1•10^6
Population
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
Year


3•10^4
2.75•10^4
2.5•10^4
2.25•10^4
2•10^4
1.75•10^4
1.5•10^4
1.25•10^4
1•10^4
7500
5000
2500
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
−2500
−5000
−7500
−1•10^4
−1.25•10^4
−1.5•10^4
−1.75•10^4
−2•10^4
−2.25•10^4
−2.5•10^4
−2.75•10^4
−3•10^4
Rate of Change of Population
Year