A. MATERIALS NEEDED

worksheet with function, calculator, and graph paper

B. OBJECTIVE

The student will discover by numerical means the limit of a function at a point and to relate the numerical information to the graph of the function.

C. RATIONALE

This activity builds on the knowledge that students have already gained concerning the evaluation of functions and plotting points. Students should understand the relationship between the function values calculated and the existence of the corresponding points on the graph of the function. The process of calculating the function values at numbers in the domain that approach the number at which the limit is being determined allows the students to see patterns which develop within the limiting process. By graphing these points, the students see the points begin to "cluster" around the limiting value of the function.
Objective: The student will discover by numerical means the limit of a function at a point and to relate the numerical information to the graph of the function.

Procedure: A function is defined below.

1. Fill in the chart with the appropriate function values.

2. Make a conjecture concerning the limit of the function based on the data that has been computed for the function and write a statement to support your conjecture.

3. Plot the points which were calculated on the graph of the function. Write a statement that describes the behavior of the graph close to \( x = 3 \).

**Function:** \( f(x) = \frac{x^2 - 9}{x - 3} \)

**Limit:** \( \lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \) ____

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GRAPH OF THE FUNCTION \( f(x) = \frac{x^2 - 9}{x - 3} \)
CRUNCHING OUT THE LIMIT

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**Function:** \( f(x) = x^2 - 4 \)

**Limit:** \( \lim_{{x \to 3}} x^2 - 4 = \ldots \)

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Function: \( f(x) = \sqrt{7 - x} \)  
Limit: \( \lim_{x \to 3} \sqrt{7 - x} = \) ________
GRAPH OF THE FUNCTION \( f(x) = \sqrt{7 - x} \)
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Function: $f(x) = 2^x$  

Limit: $\lim_{{x \to 3}} 2^x = \ldots$

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GRAPH OF THE FUNCTION

\[ f(x) = 2^x \]
**CRUNCHING OUT THE LIMIT**

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**Function:**

\[
f(x) = \begin{cases} 
  x + 2 & \text{if } x \leq 3 \\
  x^2 - 4 & \text{if } x > 3 
\end{cases}
\]

**Limit:**

\[
\lim_{x \to 3} f(x) = \ldots
\]

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CRUNCHING OUT THE LIMIT

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Function: $f(x) = \begin{cases} x^2 - 5 & \text{if } x \leq 3 \\ 2x - 3 & \text{if } x > 3 \end{cases}$

Limit: $\lim_{x \to 3} f(x) = \phantom{0}$

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