Introduction to Symbolic Mathematics

Purpose:

The purpose of this experiment is to learn how to do symbolic mathematics via Mathematica.

Equipment and Chemicals:

A PC running *Mathematica 3.0* and a printer.

Directions:

See the instructor for directions on how to load and run Mathematica.

Calculations:

Note in the following that all Mathematica functions are start with a capital letter, e.g., Sin [x], Exp [x], not sin [x] or exp [x].

Differentiation:

We can use *Mathematica* to take derivatives symbolically. The basic form of the derivative command is **D[expression,var]**. For example:

Example 1:

Input: D[Sin[x],x]

Output: $\cos(x)$

Example 2:

Input: Clear[k]

Input: D[Sin[k x],x]

Output: $k\cos(k\, x)$

To take the *n*-th derivative of *f*, use **D[f[x],{x,n}]**. To differentiate with respect to several variables for instance, you could use **D[f[x,y,z],{x,1},{y,2},{z,1}]**. This computes the following fourth partial derivative:

Example 3:

Output: $\frac{\partial^4}{\partial x \partial y^2 \partial z} f(x, y, z)$

Another example is:

Example 4:

Input: D[f[x,y,z],{x,1},{y,2},{z,1}]

Output: $f^{(1,2,1)}(x, y, z)$

Notice that the superscripts which specify the order of the partial derivative appear in the same order that the variables are passed to the function. It does not depend on the order in which the derivatives are specified. Thus, taking the *x* and *y* derivatives in the opposite order gives the same result.

Example 5:

Input: D[f[x,y,z],{y,2},{x,1},{z,1}]
Output: $f^{(1,2,1)}(x, y, z)$

INTEGRATION:

Next we consider integration. **Integrate[f[x],x]** returns the indefinite integral of f(x). The arbitrary constant associated with the indefinite integral is set to zero. **Integrate[f[x],{x,xmin,xmax}]** computes the definite integral of *f(x)* between *xmin* and *xmax*. You can do a definite multiple integral using the same syntax as for a numerical multiple integral. That is **Integrate[f[x,y],{x,xmin,xmax},{y,ymin,ymax}]** computes the multiple integral of *f(x,y)*. To compute an indefinite multiple integral you need to nest the command **Integrate** as shown below.

Example 6:

Input: Integrate[Integrate[x^2 Sin[x y],x],y]

Output: $\frac{-\cos(x\, y) - x\, y \sin(x\, y)}{y^2}$

Example 7:

Input: test=Integrate[1/(2+Cos[x]),x]

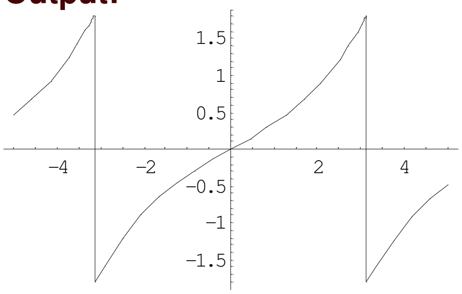
Output: $\frac{2 \tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$

We can even use *Mathematica* to plot a function. **Plot[*f*, {*x*, *xmin*, *xmax*}]** generates a plot of *f* as a function of *x* from *xmin* to *xmax*. Here we plot the function we just calculated.

Example 8:

Input: plot1=Plot[test,{x,-5,5}];

Output:



Example 9:

Input: `Integrate[E^(-a x), {x, 0, Infinity}]`

Output: $\text{If}\left[\text{Re}[a] > 0, \frac{1}{a}, \int_0^\infty e^{-a\,x} \, dx\right]$

As your laboratory assignment, use *Mathematica* to solve the following Calculus problems. Print out your results.

Differentiation:

Calculate *y'* and *y''* for each of the following:

1. $y = \ln\sqrt{x^2+1}$
2. $y = x^3 e^{4x-1}$
3. $y = \sin^4(4x)\,e^{5x^2-1}$

Consider the function *z*, such that $z = f(x,y)$. Calculate the partial derivatives $\left(\frac{\partial z}{\partial x}\right)_y$, $\left(\frac{\partial^2 z}{\partial x^2}\right)_y$, $\left(\frac{\partial^2 z}{\partial x \partial y}\right)$, and $\left(\frac{\partial^2 z}{\partial y \partial x}\right)$ for the function

$z = x^2 y^2 + y^3$.

Integration:

Calculate the following integrals.

1. $I = \int \frac{e^x}{e^x + 1}$
2. $I = \int_1^8 \sqrt[3]{x}\, dx$
3. $I = \int_{-\pi}^{\pi} \sin^5 x \cos x\, dx$
4. $I = \int_{x=-1}^{x=1} \int_{y=-\sqrt{1-x^2}}^{y=\sqrt{1-x^2}} 2\sqrt{1-x^2}\, dy\, dx$
5. $I = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=1} \sqrt{r^2+1}\, r\, dr\, d\theta$